

Optimization of Inventory Cost for Stock Dependent Demand with Normally Distributed Lead Time Considering Complete Backlogging

Md. Ashiful Alam, Zahid Hasan, Anik Ghosh, Nasib Al Habib, Marjia Haque

Abstract— Inventories are raw materials, work in process and completely finished goods that are considered to be the portion of business's assets that are ready or will be ready for sale. Formulating a suitable inventory model is one of the major concerns for an industry. Numerous researches have been done on the inventory models both for the deterministic and probabilistic situation in the last several decades. In this research inventory model has been developed and demand elasticity of product has been considered in developing an optimal order quantity and inventory cost based on the assumption. The deterministic model tells an optimum order quantity & maximum possible shortages that are completely backlogged. It has been developed by using the differential equation for inventory consumption which has been solved for different periods. Total inventory cost was derived with respect to two variables order quantity and backlogging quantity.

Keywords — Demand Rate, Lead Time, Ordered Quantity, Re-order quantity, Backlogging Cost, Holding Cost, Demand Elasticity

1 INTRODUCTION

INVENTORY or stock means to the goods and materials that a business holds for the ultimate goals to have a purpose of resale (or repair). Inventory management is a discipline and primarily about specifying the shape and placement of stock in goods. It is required at different locations within a facility or within many locations of a supply network to precede the regular and planned course of production and stock of materials. This article is concerned with an inventory model which is quite simple and standard. Attention is restricted to a simple familiar class of control policies the re-order/order quantity or (Q, R) policies. When the inventory position reaches the reorder quantity R, an order is placed for the fixed amount Q, the batch size.

Normally demand rate is assumed to be a given constant but here demand is constant up to reorder point and then the demand is uncertain. The change in demand in response to inventory or marketing decisions is commonly referred to as demand uncertainty [1]. Recently many researchers have developed a series of theories to find the optimal inventory cost such that base stock (Q, R) model.

Inventory management involves a retailer seeking to acquire and maintain a proper merchandise assortment while ordering, shipping, handling and related costs are kept in check. It also involves systems and processes that identify inventory requirements, set targets, provide replenishment techniques, reports actual and projected inventory status and handles all functions related to the tracking and management of material. This would include the monitoring of material moved into and out of stockroom locations and the reconciling of the inventory balances. It also may include ABC analysis, lot tracking, cycle counting support, etc. Management of the inventories, with the primary objective of determining/controlling stock levels within the physical distribution system, functions to balance the need for product availability against the need for minimizing stock holding and handling costs [2].

When a public company has a backlog, there can be implications for shareholders because the backlog may have an impact on the company's future earnings, as it is unable to meet demand. Orders that remain unfulfilled or unprocessed are considered backlogged orders. This research deals with holding cost, ordering cost and backlogging cost to determine the total cost where

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backlogging is used instead of penalty cost during lead time demand uncertainty.

2 BACKGROUNDS

Inventory optimization models can be either deterministic that deals with every set of variable states uniquely determined by the parameters in the model, or stochastic that coincides with variable states described by probability distributions. Debashis Dutta and Pavan Kumar derived a differential equation inventory model that incorporates partial backlogging and deterioration [2]. Holding cost and demand rate are time-dependent. Alam et al. showed the inventory optimization model by considering fill rate [3]. Taha treated unconstrained probabilistic inventory problems with constant units of cost [5]. Hadley discussed probabilistic continuous review inventory models with constant units of cost and the lead-time demand is a random variable [6]. Their work gives heuristic approximate treatment for each of the backorders and the lost sales cases. Fabrycky studied the probabilistic single-item, single source (SISS) inventory system with zero lead-time, using the classical optimization [23]. Abou-El-Ata et al. introduced a probabilistic multi-item inventory model with varying order cost; zero lead-time demand under two restrictions and no shortage are to be allowed [4]. Fergany applied several continuous distributions for constrained probabilistic lost sales inventory models with varying order cost using Lagrangian method [24]. Recently, Kotb deduced multi-item EOQ model with varying holding cost using geometric programming approach [25].

Montgomery, Bazarra et al. developed both deterministic and stochastic models considering the situation in which a fraction of demand during the stock out period is back ordered and remaining is lost forever [7]. Rosenberg developed a lot-size inventory model with partial backlogging taking factious demand rate that simplifies the analysis [8]. Many authors including Feldman, Richards, and Sahin have studied continuous review inventory models with constant units of cost and stationary distributions of inventory level [9][10][11]. The inventory models under continuous review with the stationary distribution of inventory level, or inventory position in the case of positive lead-time, have been derived using renewal theory as in Arrow [12]. In addition, Ben-Daya and Abdul examined unconstrained inventory model with constant units of cost, demand follows a normal distribution and the lead-time is one of the decision variables [13].

A deterministic inventory model is developed for deteriorating items in which shortages are allowed and salvage value is incorporated into the deteriorated items in by Vinod Kumar Mishra [14]. Setup Cost and Lead Time Reductions on Stochastic Inventory Models with a

Service Level Constraint was modeled by Chuang Hung-Chi Chang, in the Journal of the Operations Research Society of Japan [15]. An inventory model with stochastic lead-time and stochastic demand is a vast work on stochastic lead time developed by Jun Deng [16]. Analyzing a Stochastic Inventory System for deteriorating items With Stochastic Lead Time Using Simulation Modeling was developed by S. Mehdi Sajadifar in 2011 [17]. Safety Stock Positioning in Supply Chains with Stochastic Lead Times was done by David Simchi et al. [18]. A model of the impact of stochastic lead time reduction on inventory cost under order crossover was developed by Jack C. Hayya et al. [19].

Inventory modeling with stochastic lead-time and price dependent demand incorporating advance payment was developed by A.K. Maiti et al. [20]. Here Inventory model for an item is developed in a stochastic environment with price-dependent demand over a finite time horizon. Here, probabilistic lead-time is considered and shortages are allowed (if occurs). In any business, placement of an order is normally connected with the advance payment (AP). Again, depending upon the amount of AP, the unit price is quoted, i.e., price discount is allowed. Till now, this realistic factor is overlooked by the researchers. In this model, the unit price is inversely related to the AP amount. Against this financial benefit, the management has to incur an expenditure paying interest against AP. Taking these into account mathematical expression is derived from the expected average profit of the system. A closed form solution to maximize the expected average profit is obtained when demand is constant. In other cases model is solved using generalized reduced gradient (GRG) technique and stochastic search genetic algorithm (GA). Moreover, results of the models without and with advance payment are presented and solved. The numerical examples are presented to illustrate the model and the results for two models obtained from two methods are compared in different cases. Also, some parametric studies and sensitivity analyses have been carried out to illustrate the behavior of the proposed model.

In classical inventory models, the demand rate & holding cost are assumed to be constant. In reality the demand & holding cost for physical goods maybe time or stock dependent. Time also plays an important role in the inventory system; therefore, in this article, we consider that demand is stock dependent up to reorder point & holding costs an increasing function of time.

In research, it considers demand rate as a linear function of inventory before reordering point & developed an inventory model with non-zero lead time & lead time demand as a function of time. This makes the work of Rathod & Bhatwala more realistic with non – zero lead time & avoiding shortage cost [21]. Demand uncertainty

can occur at any time. So we have let shortage to occur and we will backlog it to completely get full customer satisfaction. This makes the work of Shuva & Nasib more realistic in an industrial scenario [22].

In extended work, it considers demand rate as linear function of inventory up to reorder point & random thereafter. In all the previous work either it was totally stochastic or totally deterministic. As there was no work combining both probabilistic & deterministic model; it was thought to be a scope for new work.

3.1 METHODOLOGY OF THE PROPOSED RESEARCH

In order to carry out this research work, steps those have been adopted are mentioned below:

1. A linear function for inventory up to reorder point & another constant demand rate during lead time has been considered

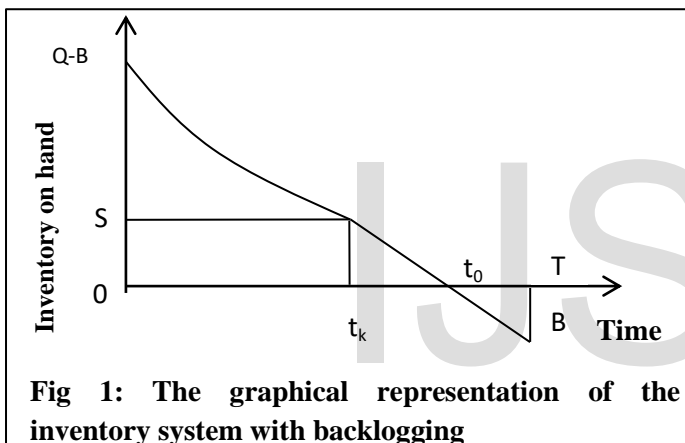


Fig 1: The graphical representation of the inventory system with backlogging

2. Using boundary condition for three different periods, Inventory with respect to time has been acquired and time for each period.
3. Holding cost of inventory was calculated by adding holding cost of each period by integrating within their time range.
4. Ordering cost was calculated by the fraction of order per day & total time.
5. The back ordering cost was calculated by integrating inventory level within their time limit.
6. Total cost was found adding holding, ordering and back ordering cost.
7. Total inventory cost was derived with respect to two variables Q & B to find out the equation of the optimum order quantity and back ordered inventory level.

TABLE 1
UNITS FOR THIS FORMULATION

Symbol	Description
Parameters	
I(t)	Inventory on hand at time t
D	Demand Rate
A	Ordering Cost
B	Demand parameter indicating elasticity in relation to the inventory
P	Shortage/Penalty Cost
h_k	Holding Cost/Item
T	Time
T	Cycle Time
TC	Total Cost
R	Re-order Quantity
Variables	
Q	Ordered Quantity
B	Backlogging Order Quantity

3.2 ASSUMPTIONS

1. Demand rate is stock dependent up to reorder point and time-dependent thereafter.
2. Replenish is not instantaneous that is Lead time is not zero and Shortages occur.
3. Demand is fulfilled after shortage with full Backlogging.
4. Order quantity varies with demand rate & lead time demand is fixed.
5. Every time we place the order of Q quantity after the stock reaches to order level S at time t=t_k.
6. Reorder point is fixed.
7. The holding cost is varying as an increasing step function of the quantity in storage or a decreasing function of time.
8. Backlogging has a positive value but considered negative for calculation.
9. The demand rate R is linear function of the inventory level up to reorder point which is expressed as $R(I(t)) = R + \beta I(t)$; $R > 0$, $0 < \beta < 1$, $I(t) \geq 0$

3.3 NOTATIONS

The following notations are used after reviewing several kinds of literature and considering some practical situations which are divided into parameters and variables in Table 1.

4 MODEL FORMULATIONS

To balance the warehouse, inventory control is too much important. It can be said in a simple sentence that, inventory mainly takes over the total stock. Cycle inventory can be used for different stages of inventory cycle. But this depends on the level of turnover.

The model starts with the inventory consumption differential equation which is stock-dependent up to reorder point & stochastic according to Rathod et.al.. Thereafter,

$$\frac{dI(t)}{dt} = -\{R + \beta I(t)\}; 0 \leq t \leq t_k; \quad (1)$$

$$= -R; t_k \leq t \leq T; \quad (2)$$

In order to solve I (t), we get from the above equation (i) by integrating both sides,

$$\int_0^t \frac{dI(t)}{R + \beta I(t)} = \int_0^t dt$$

$$\frac{1}{\beta} \ln \frac{R + \beta I(t)}{R + \beta(Q - B)} = -t$$

Using boundary condition I (0) = Q-B, we get, (1st period)

$$R + \beta I(t) = R + \beta(Q - B)e^{-\beta t}$$

$$I(t) = \frac{1}{\beta} \left[\{R + \beta(Q - B)\}e^{-\beta t} - R \right] \text{ When } (0 \leq t \leq t_k) \quad (3)$$

In order to solve I (t), we get from the above equation (2) by integrating both sides,

2nd period,

$$\int_{t_k}^t dI(t) = - \int_{t_k}^t dt$$

Using the boundary condition, I (t_k) = S, we get,

$$I(t) - S = -R(t - t_k)$$

$$I(t) = S - R(t - t_k), \text{ When } t_k \leq t \leq t_0 \quad (4)$$

3rd period,

$$I(t) = S - R(T - t), \text{ When } t_0 \leq t \leq T \quad (5)$$

At t=t_k, I (t) =S

Therefore from equation (3),

$$S = \frac{1}{\beta} \left[\{R + \beta(Q - B)\}e^{-\beta t_k} - R \right]$$

$$t_k = \frac{1}{\beta} \ln \frac{R + \beta S}{R + \beta(Q - B)} \quad (6)$$

At t=t₀, I (t) =0 from equation (4), we get,

$$t_0 = \frac{1}{\beta} \ln \frac{R + \beta(Q - B)}{R + \beta S} + \frac{S}{R} \quad (7)$$

At t=T, I (t) =-B from equation (5), we get

$$T = \frac{1}{\beta} \ln \frac{R + \beta(Q - B)}{R + \beta S} + \frac{S}{R} + \frac{S + B}{R} \quad (8)$$

The holding cost is a decreasing function of inventory with time. The holding cost can be divided into segments. The holding cost up to reorder point from start period & the holding cost from the start of lead time to zero inventory.

Therefore, Holding Cost,

$$HC = h_k \int_0^{t_k} I(t) dt + h_k \int_{t_k}^{t_0} I(t) dt$$

$$HC = \frac{h_k}{\beta^2} (R + \beta Q - \beta B) (1 - e^{-\beta t_k}) - \frac{R h_k}{\beta} t_k + h_k S (t_0 - t_k) - \frac{h_k R}{2} (t_0 - t_k)^2 \quad (9)$$

$$\text{Backlogging cost} = b \int_{t_0}^T I(t) dt$$

$$BC = b \int_{t_0}^T \{S - R(T - t_0)\} dt$$

$$BC = -bB \frac{S+B}{R} \quad (10)$$

There ordering cost of an inventory cycle = $\frac{A}{T}$

$$OC = \frac{A}{\frac{1}{\beta} \ln \frac{R + \beta(Q-B)}{R + \beta S} + \frac{S}{R} + \frac{S+B}{R}} \quad (11)$$

Finally, the total cost (TC) is the summation of holding cost, ordering cost and backlogging cost.

$$TC = \frac{A}{\frac{1}{\beta} \ln \frac{R + \beta(Q-B)}{R + \beta S} + \frac{S}{R} + \frac{S+B}{R}} + \frac{h_k}{\beta^2} (R + \beta Q - \beta B) (1 - e^{-\beta t_k}) - \frac{R h_k}{\beta} t_k + h_k S (t_0 - t_k) - \frac{h_k R}{2} (t_0 - t_k)^2 - bB \frac{S+B}{R}$$

$$TC = \frac{A}{\frac{1}{\beta} \ln \frac{R + \beta(Q-B)}{R + \beta S} + \frac{S}{R} + \frac{S+B}{R}} + \frac{h_k}{\beta^2} \{R + \beta(Q-B)\} \left\{1 + \frac{R + \beta(Q-B)}{R + \beta S}\right\} - \frac{R h_k}{\beta^2} \ln \frac{R + \beta(Q-B)}{R + \beta S} + \frac{h_k S^2}{2R} - bB \frac{S+B}{R} \quad (12)$$

For optimum order quantity, we equate $\frac{d(TC)}{dQ} = 0$

$$\frac{d(TC)}{dQ} = \frac{-A}{\left\{\frac{1}{\beta} \ln \frac{R + \beta(Q-B)}{R + \beta S} + \frac{S}{R} + \frac{S+B}{R}\right\}^2 \{R + \beta(Q-B)\}} + \frac{h_k}{\beta} + \frac{2h_k \{R + \beta(Q-B)\}}{\beta(R + \beta S)} - \frac{R h_k}{\beta \{R + \beta(Q-B)\}} = 0 \quad (13)$$

For optimum backlogging quantity, we equate $\frac{d(TC)}{dB} = 0$

$$\frac{d(TC)}{dB} = \frac{-A}{\left\{\frac{1}{\beta} \ln \frac{R + \beta(Q-B)}{R + \beta S} + \frac{S}{R} + \frac{S+B}{R}\right\}^2} \left\{ \frac{1}{R + \beta(Q-B)} - \frac{1}{R} \right\} - \frac{h_k}{\beta} - \frac{2h_k [R + \beta(Q-\beta)]}{\beta(R + \beta S)} + \frac{R h_k}{\beta [R + \beta(Q-B)]} - \frac{b}{R} (S + 2B) = 0 \quad (14)$$

5.1 MODEL IMPLEMENTATION

Table 2: Table of data analysis

Cycle	Demand Rate (per day)	Order Quantity (units)	Backloging Quantity (units)	Lead-time (days)	Cycle time (days)
1	190	367	130	1.94	2.6
2	210	407	133	1.74	2.4
3	225	435	133	1.63	2.4
4	250	481	135	1.46	2.2
5	160	303	130	2.31	2.8
6	130	235	130	2.84	3.3
7	100	157	128	3.70	3.8
8	110	183	128	3.38	3.6
9	120	223	129	2.96	3.3
10	140	260	130	2.64	3.1
11	150	293	130	2.38	2.9
12	170	326	130	2.17	2.8

Table 2 shows lead time in accordance with its demand rate of various cycles. From the analysis, we can see how order quantity, backloging quantity and lead time has changed in accordance with the variation of demand rate. Lead time & cycle time was calculated by using the formula from the model. From figure 2 it can be seen that when demand is high its tough meet the demand always. So backloging is a probable case there. It can be said that backloging is proportional to demand rate. It is supposed to get higher when demand rate is high. In the graph, it shows also proves that backloging quantity gets high when demand rate is high.

From the figure 3, it can be said that lead time is inversely proportional to demand rate. As reorder point has been considered fixed when demand rate is high lead time gets low. When demand rate is low; lead time is high as inventory tends to finish slowly. Though reorder point is fixed due to low demand rate lead time is high.

In figure 4, it shows the variation of cost with & without backloging. It is obvious that total cost of the cycle has to be low without backloging.

5.2 NUMERICAL EXAMPLE

The following numerical values of the parameter in the proper unit were considered as input for numerical analysis of the model:

$A = \$100/\text{order}$, $R = 210/\text{day}$, $S = 250$ units, $h_k = \$0.005/\text{unit/day}$
 $b = \$1/\text{unit}$;

The output of the model has been found out by solving Equations (xvi) & (xvii) through iteration process.

The optimum order quantity obtained $Q = 407$ Units & Backloging $B = 133$ Units. The cycle time, $T = 2.4669$ days
Lead time = 1.747 days

Total inventory cost for the optimum quantity = \$137.22

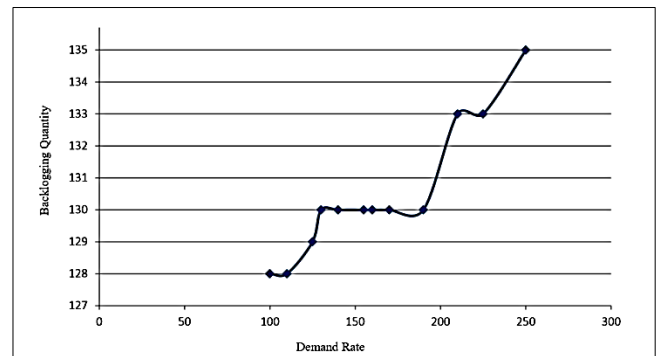


Figure 2: Backloging Quantity versus Demand Rate

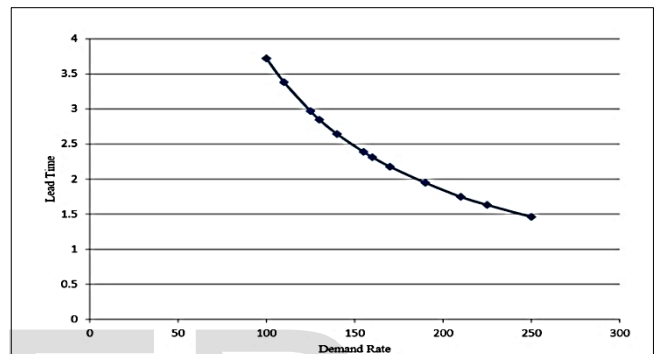


Figure 3: Lead time versus Demand Rate

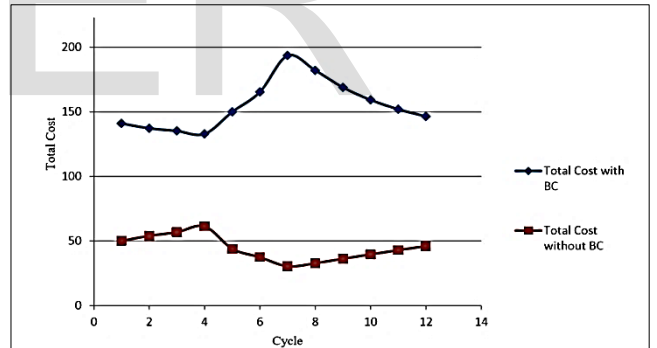


Figure 4: Total Cost with and without Backloging

The case of decreasing holding cost considered in this paper applies rented storage facilities, where lower rent rates are normally obtained for longer-term leases. In this model, a linearly decreasing holding cost has been used to find the optimal result for the order quantity and backloging quantity. That ultimately has optimized the total cost of the cycle. Complete backloging has been used to satisfy the demands of all customers as shortages occur because customers who experience stock out may not purchase the goods again from the respective suppliers and they may turn to another store to purchase the goods. So, it is necessary to take backloging into action.

6 RESULTS

The proposed model of inventory cost optimization has been successfully implemented in the previous section of this study. The objectives were obtained after the model implementations. The Total cost equation found is

$$TC = \frac{A}{\frac{1}{\beta} \ln \frac{R + \beta(Q-B)}{R + \beta S} + \frac{S}{R} + \frac{S+B}{R}} + \frac{h_k}{\beta^2} \{R + \beta(Q-B)\} \left\{1 + \frac{R + \beta(Q-B)}{R + \beta S}\right\} - \frac{R h_k}{\beta^2} \ln \frac{R + \beta(Q-B)}{R + \beta S} + \frac{h_k S^2}{2R} - bB \frac{S+B}{R} \quad (12)$$

7 FUTURE SCOPES

Future researches can be done in this study which is given below,

1. The deterministic model can be extended to accommodate planned shortages, variable ordering costs, and non-instantaneous receipt of orders, incorporation of deteriorating items, shortage cost, partial backlogging etc. to develop more complex models.
2. This model can also be developed by removing the constraint which is fixed to reorder point and make it variable. An equation can be developed with respect to R which can incorporate with our model that will also determine an optimum reorder point for different demand rate with order quantity & backlogging quantity.
3. The solution of the stochastic model will be done to get an optimum order quantity & reorder point. Then that result will be used get the optimum total cost for the cycle.

8 CONCLUSIONS

In this research, an inventory model is developed with stock-dependent demand rate, and complete backlogging with time-varying holding Cost with non-instantaneous receipt of orders. Shortages are allowed that has to be backlogged.

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